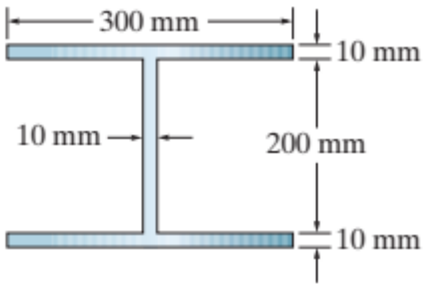
Q1. A rubber ball is inflated to a pressure of 60 kPa. At that pressure the diameter of the ball is 230 mm and the wall thickness is 1.2 mm. The rubber has modulus of elasticity E = 3.5 MPa and Poisson's ratio v = 0.45. Determine the maximum stress and strain in the ball.

Q2. A thin cylindrical pressure vessel has an internal diameter of 150 mm and a wall thickness of 5 mm. It is subjected to an internal pressure of 7 N/mm^2. If the cylinder is 900 mm long and E = 200 GPa, find the value of Poisson's ratio for the material if the change in volume under this pressure is 15,500 mm^3.

Q3. A 2014-T6 aluminum alloy column has a length of 6 m and is fixed at one end and pinned at the other. If the cross-sectional area has the dimensions shown, determine the critical load. σ_Y = 250 MPa.



Q4. A steel column is constructed from an I section, with the following properties $I_{xx} = 160x10^6$ mm^4 and $I_{yy} = 11.1x10^6$ mm^4. Determine the critical load if its bottom end is fixed supported and its top is free to move about the strong axis and is pinned about the weak axis.

Q5. A thick-walled, closed-ended cylinder of inner radius a and outer radius b is subjected to an internal pressure p_i only. The cylinder is made of a material with permissible tensile strength σ_{all} and shear strength τ_{all}. Calculate the allowable value of p_i. Given: a = 0.8 m, b = 1.2 m, σ_{all} = 100 MPa, τ_{all} = 60 MPa.

① Given:

pressure inside ball (P) = 60 KPa
Diameter of ball (D) = 230 mm
wall thickness (t) = 1.2 mm
modulus of elasticity of rubber (E) = 3.5 MPa
Poisson's Ratio (μ) = 0.45

$$\text{Maximum Stress} = \frac{PD}{4t}$$

$$= \frac{60 \times 10^{-3} \times 230}{4 \times 1.2}$$

$$= \boxed{2.875 \ N/mm^2}$$

$$\text{Maximum strain in ball} = \frac{\sigma_n}{E} - \frac{\mu \sigma_n}{E}$$

$$= \frac{2.875}{3.5} - 0.45 \times \frac{2.875}{3.5}$$

$$= 0.8214 - 0.3696$$

$$= \boxed{0.4518}$$

② Given:

Internal diameter (d) = 150 mm
Wall thickness (t) = 5 mm
Internal pressure (p) = $7 N/mm^2$
Length of cylinder (L) = 900 mm
Young's Modulus (E) = 200 GPa = $200 \times 10^3 N/mm^2$
Change in Volume ΔV = 15,500 mm^3

$$\Rightarrow \text{Hoop stress } (\sigma_h) = \frac{pd}{2t} = \frac{7 \times 150}{2 \times 5} = 105 \ N/mm^2$$

$$\text{Longitudinal Stress } (\sigma_L) = \frac{pd}{4t} = \frac{7 \times 150}{4 \times 5} = 52.5 \ N/mm^2$$

Now

$$\text{Hoop strain} \quad \epsilon_h = \frac{\sigma_h}{E} - \nu \cdot \frac{\sigma_L}{E}$$

$$= \frac{105}{200 \times 10^3} - \nu \frac{52.5}{200 \times 10^3} = 0.000525 - \nu \cdot 0.0002625$$

$$\text{Longitudinal strain} \quad \epsilon_L = \frac{\sigma_L}{E} - \nu \cdot \frac{\sigma_h}{E}$$

$$\epsilon_L = \frac{52.5}{200 \times 10^3} - \nu \frac{105}{200 \times 10^3} = 0.0002625 - \nu \cdot 0.000525$$

Initial Volume $\Rightarrow V = \pi r^2 L \quad \left[r = \frac{d}{2} = 75\,mm \right]$

$$= (3.14)(75)^2 \times 900$$

$$= 15896250 \ mm^3$$

$$\Delta V = V \cdot (2 \cdot \epsilon_n + \epsilon_e)$$

$$15500 = 15896250 \times (0.000525 - \nu \cdot 0.0002625) + (0.0002625 - \nu \cdot 0.000525)$$

$$0.000975 = (0.000525 - \nu \cdot 0.0002625) + (0.0002625 - \nu \cdot 0.000525)$$

$$0.000975 = (0.0007875 - \nu \cdot 0.0007875)$$

$$0.0001875 = -\nu \cdot 0.0007875$$

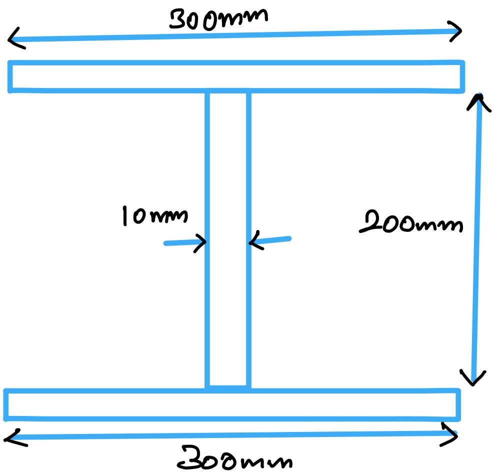
$$\boxed{\nu = -0.238}$$

③ Determine the critical load

$$\sigma_y = 250 \ MPa$$

$$E_A = 73.1 \ GPa$$ ← 2014-T6 Aluminium

$$L = 6 m$$



$$I_x = \frac{10 \times 200^3}{12} + \frac{1}{12}(300)(220^3 - 200^3)$$

$$= 72866666.6667 \ mm^4$$

$$I_y = \frac{10^3 \times 200}{12} + \frac{1}{12}(300)^3(220 - 200)$$

$$= 45016666.6667 \ mm^4$$

$$I_y \leq I_x$$

Crippling Load : $F_{cr} = \frac{\pi^2 E I}{Le^2}$

$$= \frac{\pi^2 \times 73.1 \times 10^3 \times 45016666.6667}{(0.7 \times 8000)^2}$$

$$\boxed{F_{cr} = 1035.65 \ KN}$$

④ MOI about Strong axis $I_{xx} = 160 \times 10^6\ mm^4$

MOI about weak axis $I_{yy} = 11.1 \times 10^6\ mm^4$

$E = 210\ GPa = 210 \times 10^3\ MPa$

Effective length along I_{xx} :

Since the column's bottom end is fixed and its top is free, its effectively a 'cantilever' column in the strong axis direction. Thus $L_{eff,xx} = 2L$

Effective length along I_{yy} :

Since the column's bottom end is fixed and the top end is pinned the effective length

$L_{eff,yy} = 0.7L$

=> Euler Buckling formula : $P_{cr} = \dfrac{\pi^2 E I}{(L_{eff})^2}$

$$\Rightarrow P_{cr,xx} = \frac{\pi^2 \times 210 \times 10^3 \times 160 \times 10^6}{(2L)^2} = 82.9 \times 10^{12}\ N/mm^2$$

$$\Rightarrow P_{cr,yy} = \frac{\pi^2 \times 210 \times 10^3 \times 11.1 \times 10^6}{(0.7L)^2} = 46.95 \times 10^{12}\ N/mm^2$$

Overall Critical Load = $P_{cr} = \min(P_{cr,xx}, P_{cr,yy})$

$$\boxed{P_{cr} = 46.95 \times 10^{12}\ N/mm^2}$$

⑤

$a = 0.8m$

$b = 1.2m$

diameter $= 2a$

$= 1.6m$

thickness $= b - a$

$= 0.4m$

$\sigma_{all} = 100\ MPa$

$\tau_{all} = 60\ MPa$

Using Lame's Equation

$$t = \frac{dia}{2}\left[\sqrt{\frac{(100 \times 10^6) + P_i}{(100 \times 10^6) - P_i}} - 1\right]$$

$$0.4 = \frac{1.6}{2}\left[\sqrt{\frac{(100 \times 10^6) + P_i}{(100 \times 10^6) - P_i}} - 1\right]$$

$$(100 \times 10^6) + P_i = 2.25\,(100 \times 10^6 - P_i)$$

$$(2.25 + 1) P_i = (225 - 100) \times 10^6$$

$$\Rightarrow P_i = 38.46 \times 10^6\ Pa$$

$$\boxed{P_i = 38.46\ MPa}$$